TOPICS IN PROBABILITY THEORY AND STOCHASTIC PROCESSES
Home Work 9 due on Monday April 15
Instructor: Prof. Artem Zvavitch

Please, use equations for limiting probabilities we discussed in class to solve following problems:

Problem 1. *Consider a job shop which consists of M machines and one serviceman. Suppose that the amount of time each machine runs before before braking down is exponentially distributed with mean $1/\lambda$ and suppose that the amount of time that it takes for serviceman to fix the machine is exponentially distributed with mean $1/\mu$.*

- *What is the average number of machines not in use?*
- *What proportion of time is each machine in use?*

Problem 2. *Consider M/M/1 queu (i.e. all $\lambda_n = \lambda$ and $\mu_n = \mu$) study the limiting probabilities P_n (note that they may not exist for some values of λ and μ!)*

Problem 3. *Consider a shoe-shop consisting of two chairs. A customer upon arrival goes initially to chair 1 where his shoes are cleaned and polish is applied. After this is done the customer moves on to chair 2 where the polish is buffed. The service time at two chairs is assumed to be independent exponentially distributed r.v. with means $1/\mu_1$ and $1/mu_2$. Suppose that potential customers arrive in accordance with a Poisson process having rate λ, a potential customer will only enter the system if both chairs are empty. Note that we may have no more than ONE customer in the shop at even given time. Thus we may model this shop as a continuous Markov chain with states 0 (no customers), 1 (customer in chair 1) and 2 (customer in chair 2). Please, find v_0, v_1, v_2 and $P_{i,j}$. Please, determine the proportion of time the process is in each of the states.*