

Graph Theory and Combinatorics MATH-42021/52021.

Instructor: Prof. Artem Zvavitch

Exam 2

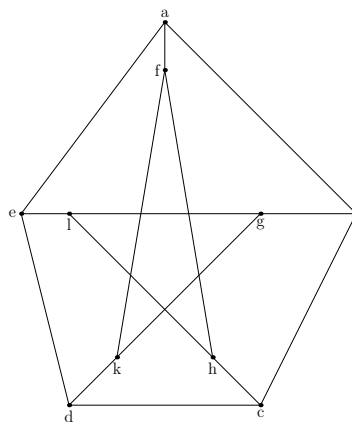
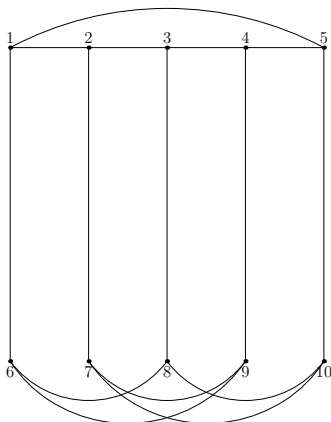
You may use ANY of your written notes/Home Works. Please, do not use books/computers/ friends. You may and should use theorems/fact/results of exercises that was proved/presented/done in class/Home Work, please stay clearly what you are using! Please, write all details of your solutions.

Problem 1. (10 pts) *There are 27 faculty (7 female and 20 male) working at the Department of Mathematical Sciences. Each year a FAC committee of 5 people is created at random. What is the probability that the number of female participants is greater than the number of male participants and Mr. Zvavitch is not a member of the committee (You do not need to compute an exact number, it is enough to write the formula).*

Problem 2. *Please,*

- **(10 pts)** *For each natural number n give an example of a graph G , with n -vertices, such that $\chi(G)\chi(\bar{G}) = n$.*
- **(20 pts)** *For each natural number n , which is a perfect square (i.e. $n = k^2$) give an example of a graph G , with n -vertices, such that $\chi(G) + \chi(\bar{G}) = 2\sqrt{n}$. (I would do this problem only when finished with all other! I would try the case $n = 4$ and $n = 9$ first)*

Problem 3. (20 pts) *Find chromatic numbers for each of the following graphs*



Problem 4. *Please,*

- **(10 pts)** *Prove that $\chi(T) = 2$, for any tree T with more than one vertex. Is it true that if $\chi(T) = 2$, then T is a tree?*
- **(10 pts)** *Also show that all trees are planar. Is it true that any planar graph is a tree?*

Problem 5. (40 pts) *Consider graph G , show that the following conditions are equivalent:*

- *G has a unique path from the root to each vertex.*
- *G has no circuits.*
- *Removal of any edge disconnects G .*