

Analytic Geometry and Calculus III (22005 Section 002)
EXAM FOR FUN
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Problem 1. Let $\mathbf{r}(t) = (2t, \sin 6t, \cos 6t)$.

- Find $\lim_{t \rightarrow 0} \mathbf{r}(t)$.
- Find $\mathbf{r}'(t)$.
- Find unit vectors $\mathbf{T}(t)$, $\mathbf{N}(t)$ and $\mathbf{B}(t)$.
- Find an equation of a tangent line to the curve defined by $\mathbf{r}(t)$ at the point $(2\pi, 0, 1)$.
- Find $\int_0^{\pi} \mathbf{r}(t)$.
- Find the curvature $k(t)$.
- Find the velocity, acceleration and speed of a particle with the position function $\mathbf{r}(t)$.
- Find the tangential and normal components of the acceleration vector.

Problem 2. Let $f(x, y) = \cos yx + x^2e^y$. Please, find

- f_x ,
- $\frac{\partial f}{\partial y}$,
- f_{yy} ,
- Check that f_{xy} .
- Check that f_{yx} .

Problem 3. Find the direction in which the function $f(x, y) = xy + xe^y$ decreases faster at the point $(1, 0)$.

Problem 4. Find equation of the tangent plane to the surface

$$x^3 - 3xy^2z + y^4 + z^3 = 0$$

at point $(1, 1, 1)$.

Problem 5. Find, and characterize the critical points of the function

$$f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$$