Introduction to Analysis
Exam for fun 2
Instructor: Prof. Artem Zvavitch
Extra 20 points, if submitted on the day of Second exam!

Problem 1. *Let* x_n *be a sequence of real numbers such that*

$$|x_{n+1}| \leq \frac{1}{3}|x_n|.$$

Show that x_n *is a convergent sequence.*

Problem 2. *Let* $x_1 = 0$ *and* $x_{n+1} = \frac{1}{10}(x_n + 2)$ *show that a)* x_n *is bounded. b) Monotone (increasing) c) convergent. Finally find the limit of* x_n.

Problem 3. *Let* $a_1 = -\frac{1}{2}$ *and*

$$a_n = -\frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots + \frac{(-1)^n}{2^n}, \text{ for } n \in N.$$

Show that a_n *is a convergent sequence.*

Problem 4. *Let* x_n *be unbounded sequence of negative numbers. Is it true that* $\lim_{n\to\infty} x_n = -\infty$*? Show that there exits a subsequence* x_{n_k} *so that* $\lim_{k\to\infty} x_{n_k} = -\infty$*?*

Problem 5. *Show that*

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

is convergetnt, also try to find the numerical value for this sum.

Problem 6. *Show that*

$$\sum_{n=1}^{\infty} \cos n$$

is divergent but

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$$

is convergent.

Problem 7. *Use Cauchy Condition to show that the series* $\sum \frac{1}{n \log\log n}$ *is convergent/divergent.*

Problem 8. *Use* $\epsilon - \delta$ *definition of continuity to show that the function* $\frac{x}{x-3}$ *is continuous at point* 4.

Problem 9. *Show that the equation* $x^5 + 7x^4 + 3x^3 - 1 = 0$ *has at least one solution on the interval* $[0, 1]$.