

Analysis 1 (42001/52001)

Exam for fun 2

Instructor: Prof. Artem Zvavitch

Extra 20 pts, if submitted on: Friday, November 7!

Problem 1. Check if sequences are convergent or divergent (you may use ANY results from class/book)

- (1) $(1 + \frac{1}{n})^{n^2}$.
- (2) $(1 + \frac{1}{n^3})^n$.
- (3) $(1 - \frac{1}{2n})^{-7n}$.

Problem 2. Let x_n be a sequence of real numbers such that

$$|x_{n+1}| \leq \frac{1}{3}|x_n|.$$

Show that x_n is a convergent sequence.

Problem 3. Let $x_1 = 0$ and $x_{n+1} = \frac{1}{10}(x_n + 2)$ show that a) x_n is bounded. b) Monotone (increasing) c) convergent. Finally find the limit of x_n .

Problem 4. Let $a_1 = -\frac{1}{2}$ and

$$a_n = -\frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \dots + \frac{(-1)^n}{2^n}, \text{ for } n \in \mathbb{N}.$$

Show that a_n is a convergent sequence.

Problem 5. Assume x_n is a bounded increasing sequence of real numbers. Show that $\lim x_n = \sup x_n$.

Problem 6. Let x_n be unbounded sequence of negative numbers. Is it true that $\lim_{n \rightarrow \infty} x_n = -\infty$? Show that there exists a subsequence x_{n_k} so that $\lim_{k \rightarrow \infty} x_{n_k} = -\infty$?

Problem 7. Use $\varepsilon - \delta$ -definition to evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1+x}}{x}.$$

Problem 8. Evaluate

$$\lim_{x \rightarrow 0} x 2^x \sin \frac{1}{x}.$$

Please, do not forget to provide explanation!! As always - you may use ANY results from class/book!

Problem 9. Use the definition of limit to show that

$$\lim_{x \rightarrow \infty} x + \cos \frac{1}{x} = \infty.$$