Introduction to Analysis 1(42001/52001)
Exam for FUN 1
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Problem 1. *Prove, using the principal of mathematical induction, that*

$$\sum_{k=1}^{n} \frac{a-1}{a^k} = 1 - \frac{1}{a^n}, \text{ for all } a \neq 0.$$

Problem 2. *Let S be a set of non overlaping squares of size at least 1 in R^2. Show that S is contable.*

Problem 3. *Solve $|x-1| \leq x+3$.*

Problem 4. *Find (and provide a proof) the supremum and infimum of $(-1,1)$.*

Problem 5. *It it true that*

$$\sup_{x \in D} f^2(x) = \left(\sup_{x \in D} f(x) \right)^2 ?$$

Please, provide a proof if the answer is yes, or counterexample if the answer is no.

Problem 6. *Assume a_n is a convergent sequence with limit a, show that then $b_n = |a_n|$ is also a convergent sequence with limits $|a|$.*

Problem 7. *Prove that*

$$\lim_{n\to\infty} \frac{\cos(\pi n)}{n} = 0.$$