Analysis 1 (42001/52001)
Home Works 10 and 11 (40 points), due on Wednesday, December 3.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Give an example of function f which is continuous but not uniformly continuous on the interval* $(2, 3)$.

Problem 2. *Give an example of a function which is continuous on* $I = (0, 1)$*, but unbounded on* I*. Prove that ANY UNIFORMLY continuous function on* I *is bounded.*

Problem 3. *Show that there exists a uniformly continuous function which is not a Lipschitz function.*

Problem 4. *Assume* $f(x)$ *is uniformly continuous on* $(0, 1)$ *such that* $f(x) > 1$ *for all* $x \in (0, 1)$*. Prove that* $1/f(x)$ *is also uniformly continuous on* $(0, 1)$.

Problem 5. *If* f *and* g *are increasing function on* $[a, b]$ *what can you say about* $f + g$*?* $f \cdot g$*?*

Problem 6. *Consider* $f : [0.1] \to \mathbb{R}$ *such that* f *is continuous,* $f(0) < f(1)$ *and* $f(x) \neq f(y)$ *for all* $x \neq y \in [0, 1]$*. Prove that then* f *is increasing.*

Problem 7. *Consider* $f : [0.1] \to \mathbb{R}$ *such that* f *is continuous and* f *has an absolute maximum at point* $c \in (0, 1)$*. Show that* f *is not a strictly monotone function.*

Problem 8. *Give an example of function* $f(x)$ *on interval* $[0, 1]$*, which is discontinuous for all irrational values* x*. Is there exists an example of such function such that* f *is decreasing? (EXPLAIN YOUR ANSWER!)*