

**Analysis 1(42001/52001)**  
**Home Work 2, due on Friday September 5.**  
**Instructor: Prof. Artem Zvavitch.**

**Problem 1.** *Let  $A$  and  $B$  be countable sets. Show that  $A \cap B$  is countable.*

**Problem 2.** *Consider a set  $S$  whose elements are non-overlapping intervals of length 1 (i.e. for any  $[a, b] \in S$  and  $[d, c] \in S$  we have that  $b - a = c - d = 1$  and  $[a, b] \cap [d, c] = \emptyset$ ). Please, show that  $S$  is a countable set.*

**Problem 3.** *Let  $S$  be a set of real numbers  $x$  such that  $x$  is a solution of a quadratic equation with integer coefficients, i.e.*

$$S = \{x \in \mathbb{R} : nx^2 + mx + k = 0 \text{ for some } n, m, k \in \mathbb{Z}\}.$$

*Proof that  $S$  is countable.*

**Problem 4.** *Consider  $a, b, c \in \mathbb{R}$ . Please, prove that*

- $a^2 + b^2 = 0$  if and only if  $a = b = 0$ .
- If  $\frac{1}{ab} > 0$  then either  $a > 0$  and  $b > 0$  or  $a < 0$  and  $b < 0$ .
- If  $b > 0$  and  $c > 0$  then  $\sqrt{b+c} > \sqrt{c}$ .
- For any positive numbers  $a, b, c, d$  if  $a < b$  and  $c < d$ , then  $ac < bd$ .
- Please, prove that  $1.4 < \sqrt{2} < 1.5$ .
- **Triangle inequality:**  $|a + b| \leq |a| + |b|$ .