Analysis 1 (42001/52001)
Home Work 3, due on Friday September 12.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Solve (please, show and explain ALL steps)*

- $-2x + 7 > x - 1$.
- $\frac{x^2-2x+1}{x-7} > 0$.
- $|x + 1| \leq |x - 7|$.

Problem 2. *Prove that*

$$2^n \geq n^2,$$

for all natural numbers $n \geq 5$.

Problem 3. *Prove that*

$$\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n} \leq 2\sqrt{n},$$

for all $n \in \mathbb{N}$.

Problem 4. *Find the infimum and the supremum of* $E = \{x \in \mathbb{R} : x = \frac{1}{n} + (-1)^n, \text{ for all } n \in N\}$.

Problem 5. *Consider an nonempty set* $E \subset \mathbb{R}$. *Prove that* E *has a supremum if and only if* $-E = \{x \in \mathbb{R} : -x \in E\}$ *has an infimum, in which case*

$$\inf(-E) = -\sup(E).$$

Problem 6. *Suppose that* f *and* g *are real valued functions with common domain* $D \subset \mathbb{R}$. *PLEASE, check the folloving statements (if it is true then prove it, if it is false then show a counterexample)*

- $\sup_{x\in D} f(x) + \sup_{x\in D} g(x) \leq \sup_{x\in D}(f(x) + g(x))$.
- $\sup_{x\in D} f(x) + \sup_{x\in D} g(x) = \sup_{x\in D}(f(x) + g(x))$.
- $\sup_{x\in D} f(x) \times \sup_{x\in D} g(x) \leq \sup_{x\in D}(f(x) \times g(x))$.
- $\inf_{x\in D} f(x) \times \inf_{x\in D} g(x) \leq \inf_{x\in D}(f(x) \times g(x))$.