Analysis 1 (42001/52001)
Home Work 4, due on Friday September 19.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Prove, using the principal of mathematical induction, that*

$$\sum_{k=1}^{n} \frac{a-1}{a^k} = 1 - \frac{1}{a^n}, \text{ for all } a \neq 0.$$

Problem 2. *Let S be a set of non overlaping squares of size at least 1 in R^2. Show that S is contable.*

Problem 3. *Show that $f(x) = \frac{2^x - x^2 + x^5}{x+5}$ is a bounded function on $[0, 1]$.*

Problem 4. *Find (and provide a proof) the supremum and infimum of $(-1, 1)$.*

Problem 5. *Find (and provide a proof) the supremum and infimum of $S = \{\frac{1}{n^2} : n \in N\}$.*

Problem 6. *It it true that*

$$\sup_{x \in D} f^2(x) = \left(\sup_{x \in D} f(x)\right)^2 ?$$

Please, provide a proof if the answer is yes, or counterexample if the answer is no.

Problem 7. *Is it true that*

- *The sum of two rational numbers is rational?*
- *The product of two rational numbers is rational?*
- *The sum of two irrational numbers is again irrational?*
- *The product of two irrational number is again irrational?*

Please, provide a proof if the answer is yes, or counterexample if the answer is no.

Problem 8. *Let $S = (0, 1) \setminus Q$, where Q is the set of rational numbers. Prove that S is uncountable.*

Problem 9. *Prove that*

$$\bigcap_{i=1}^{\infty} (0, \frac{1}{n}) = \emptyset.$$