Analysis 1 (42001/52001)
Home Work 8, due on Wednesday, November 12.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Use sequential criterion to show that*

$$\lim_{x\to 0} \frac{1}{x^2}$$

does not exist.

Problem 2. *Use sequential criterion to show that*

$$\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$$

does not exist.

Problem 3. *Use $\varepsilon - \delta$-definition of limit to show that*

$$\lim_{x\to 0} x\cos\frac{1}{x} = 0.$$

Problem 4. *Use $\varepsilon - \delta$-definition of limit to show that*

$$\lim_{x\to c} x^4 = c^4.$$

Problem 5. *Use $\varepsilon - \delta$-definition of limit to show that if*

$$\lim_{x\to c} f(x) = L$$

then

$$\lim_{x\to c} f(x)^2 = L^2.$$

Show that converse statement is no longer true.

Problem 6. *Suppose that f and g has a limit as $x \to \infty$ and that $f(x) \leq g(x)$ for all $x \in (15, \infty)$. Prove that*

$$\lim_{x\to\infty} f(x) \leq \lim_{x\to\infty} g(x).$$

Problem 7. *Use $\varepsilon - \delta$ definition to show the continuity of function $f(x) = \frac{1}{x+1}$ at point $x = 2$.*

Problem 8. (Extra 10pts) *Find, learn and write down geometric proof that*

$$\lim_{x\to 0} \frac{\sin x}{x} = 1.$$