Analysis 1 (42001/52001)
Home Work 9, due on Friday, November 21.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Use* $\varepsilon - \delta$ *definition to show the continuity of function* $f(x) = \frac{1}{x+1}$ *at point* $x = 2$.

Problem 2. *Suppose that* f *is a continuous function on* $[0, 1]$ *such that* $f(r) = 0$ *for every rational number* r. *Prove that* $f(x) = 0$ *for all* $x \in [0, 1]$.

Problem 3. *Suppose that* f *and* g *are continuous function on* $[0, 1]$ *such that* $f(r) = g(r)$ *for every rational number* r. *Prove that* $f(x) = g(x)$ *for all* $x \in [0, 1]$.

Problem 4. *Show that the function*

$$f(x) = \frac{\sqrt{x+1}}{\sqrt{1+\sqrt{x}}}$$

is a continuous function for $x \in [0, \infty)$.

Problem 5. *Show an example of bounded set* A *and a function* f *continuous on* A, *such that* f *is unbounded function on* A.

Problem 6. *Prove that if*

$$|f(x) - f(y)| \le |x - y|^2$$

for all x, y *in* $[0, 1]$ *then* $f(x)$ *is a continues function on* $[0, 1]$.

Problem 7. *Consider a continuous function* $f : [0, 1] \to R$, *such that* $f(x) > 0$, *for all* $x \in [0, 1]$. *Prove that there exists a number* $\alpha > 0$ *such that* $f(x) > \alpha$.

Problem 8. *Show that function* $f(x) = 2 \ln x + \sqrt{x} - 2$ *has a root in the interval* $[1, 2]$.

Problem 9. *Consider a continuous function* $f : [0, 1] \to [0, 1]$, *show that the equation* $f(x) = x$ *has at least one solution.*

Problem 10. *Give an example of a continuous function* $f : [0, \infty) \to [0, \infty)$, *such that the equation* $f(x) = x$ *has no solutions.*