

**Functions of Complex Variables 1**  
**Home Work 10, due on Wednesday April 4.**

**Problem 1.** Prove that  $e^{z/2}z \prod_{n=1}^{\infty} (1 + \frac{z^2}{4n^2\pi^2})$  is the Hadamard product for  $e^z - 1$ .

**Problem 2.** Show that if  $f$  is entire function of finite order that omits two values, then  $f$  is a constant. **Hint:** start with proving that if  $f$  misses  $a$ , then  $f(z) - a$  is of the form  $e^{p(z)}$ , where  $p$  is a polynomial.

**Problem 3.** Use Hadamard's theorem to show that the equation  $e^z - z = 0$  has infinitely many solutions in  $\mathbb{C}$ .

**Problem 4.** We say that  $f : U \rightarrow V$  is a local bijection, i.e. for any  $z \in U$  there exists an open disc  $D \subset U$  such that  $f : D \rightarrow f(D)$  is a bijection. Show that a holomorphic map  $f : U \rightarrow V$  is a local bijection **if and only if**  $f'(z) \neq 0$  for all  $z \in U$ .

**Problem 5.** Is it true that if  $f : U \rightarrow V$  is holomorphic with  $f'(z) \neq 0$ , then  $f$  is injective?