

**Functions of Complex Variables 1**  
**Home Work 13, due on Wednesday April 25.**

**Problem 1.** Consider  $\lambda \in \mathbb{R} \setminus \{1\}$ . Show that the function

$$S(z) = \int_0^z \frac{d\xi}{\sqrt{\xi(\xi-1)(\xi-\lambda)}}$$

maps the upper half-plane conformally to a rectangle, one of whose vertices is the image of the point at infinity. In particular show that when  $\lambda = -1$  we get a square as the image.

**Problem 2.** Let  $P$  be a simply connected region bounded by a polygon with vertices  $a_1, \dots, a_n$  and angles  $\lambda_1\pi, \dots, \lambda_n\pi$  and  $F$  is a conformal map of the disc  $\mathbb{D}$  to  $P$ , then there exist complex numbers  $B_1, \dots, B_n$  on the unit circle, and constant  $c_1, c_2 \in \mathbb{C}$  so that

$$F(z) = c_1 \int_1^z \frac{d\xi}{(\xi - B_1)^{\beta_1} \dots (\xi - B_n)^{\beta_n}} + c_2.$$

**Problem 3.** Show that if

$$F(z) = \int_1^z \frac{d\xi}{(1 - \xi^n)^{2/n}},$$

then  $F$  maps  $\mathbb{D}$  conformally onto the interior of a regular polygon with  $n$  sides and find the perimeter of this polygon.