Functions of Complex Variables 1
Home Work 3, due on Wednesday February 7.

Problem 1. *Expand*

- $\frac{1}{(1-z)^m}$, $m \in \mathbb{N}$ *in powers of* z.
- $\frac{2z+3}{z+1}$ *in powers of* $z-1$.

What is the radius of convergence of the second one?

Problem 2. *For what values of* $z \in \mathbb{C}$ *is*

- $\sum\limits_{n=0}^{\infty} \left(\frac{z}{1+z}\right)^n$ *is convergent?*
- $\sum\limits_{n=0}^{\infty} \frac{z^n}{1+z^{2n}}$ *is convergent?*

Problem 3. *Find the values of* $\sin i$, $\cos i$ *and* $\tan(1+i)$.

Problem 4. *Let* γ *be a smooth curve in* $\mathbb{C}$ *parametrized by* $z(t) : [a,b] \to \mathbb{C}$. *Let* γ^- *denote the curve with the same image as* γ *but with the reverse orientation. Prove that for any continuous function* f *on* γ

$$\int_{\gamma} f(z)dz = -\int_{\gamma^-} f(z)dz.$$

Problem 5. *Compute* $\int_{\gamma} Re(z)dz$, *where* γ *is the direct line segment from* 0 *to* $1+i$.

Problem 6. *Consider an analytic function* $f : \Omega \to \mathbb{C}$ *and a closed curve* $\gamma \subset \Omega$. *Please, show that*

$$\int_{\gamma} \overline{f(z)} f'(z)dz$$

is purely imaginary.

Problem 7. *Let us define* $\int_{\gamma} f(z)\overline{dz} = \overline{\int_{\gamma} \bar{f}(z)dz}$. *Please, compute*

$$\int_{\gamma} z^n \overline{dz}$$

for each natural $n = 0, 1, 2, \ldots$ *and* $\gamma = \{z : |z-a| = R\}$ *for a fixed* $a \in \mathbb{C}$ *and* $R > 0$. *No consider a polynomial* $P(z)$ *and compute*

$$\int_{\gamma} P(z)\overline{dz},$$

where $\gamma = \{z : |z-a| = R\}$ *for a fixed* $a \in \mathbb{C}$ *and* $R > 0$.