Functions of Complex Variables 1
Home Work 9, due on Wednesday March 21.

Problem 1. *Find the order of growth of*

- $P(z)$, *where* p *is polynomial.*
- $e^{P(z)}$, *where* $P(z)$ *is a polynomial of degree* n.
- e^{e^z}.

Problem 2. *Prove that*

$$\prod_{n=1}^{\infty}(1+\frac{z}{n})e^{-z/n},$$

converges absolutely and uniformly on every compact set.

Problem 3. *Prove that the value of an absolutely convergent product does not change if the factors are reordered.*

Problem 4. *Provet that*

$$\frac{\pi}{2}=\frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdots\frac{2m\cdot 2m}{(2m-1)\cdot(2m+1)}\cdots$$

Problem 5. *Use some trigonometry to prove that for every* z *the following product converges and prove the equality*

$$\prod_{k=1}^{\infty}\cos(\frac{z}{2^k})=\frac{\sin z}{z}$$

Problem 6. *Prove that if* $|z|<1$, *then*

$$(1+z)(1+z^2)(1+z^4)\cdots=\prod_{k=0}^{\infty}(1+z^{2^k})=\frac{1}{1-z}.$$

Problem 7. *Study the following properties of infinite products*

- *Show that if* $\sum |a_n|^2$ *converges then the product* $\Pi(1+a_n)$ *converges to a non-zero limit if and only if* $\sum a_n$ *converges.*
- *Find an examle of a sequence of complex numbers* $a_{n}{}_{n=1}^{\infty}$ *such that* $\sum a_n$ *converges but* $\Pi(1+a_n)$ *diverges.*
- *Find an example such that* $\Pi(1+a_n)$ *converges and* $\sum a_n$ *diverges.*