

Introduction to Analysis
Home Work 10, due Thursday, December 4.
Instructor: Prof. Artem Zvavitch

Problem 1. Give an example of function f which is continuous but not uniformly continuous on the interval $(2, 3)$.

Problem 2. Give an example of a function which is continuous on $I = (0, 1)$, but unbounded on I . Prove that ANY UNIFORMLY continuous function on I is bounded.

Problem 3. Show that there exists a uniformly continuous function which is not a Lipschitz function. **Hint:** consider $f(x) = \sqrt{x}$ on the interval $[0, 1]$, do not forget to prove your claim!

Problem 4. If f and g are increasing function on $[a, b]$ what can you say about $f + g$? $f \cdot g$?

Problem 5. Consider $f : [0, 1] \rightarrow \mathbb{R}$ such that f is continuous, $f(0) \leq f(1)$ and $f(x) \neq f(y)$ for all $x \neq y \in [0, 1]$. Prove that then f is increasing.

Problem 6. Give an example of function $f(x)$ on interval $[0, 1]$, which is discontinuous for all irrational values x . Is there exists an example of such function such that f is decreasing? (EXPLAIN YOUR ANSWER!)