

Introduction to Analysis 1(42001/52001)
HW1, due Wednesday, September 7 (or if needed Monday
September 12)
Instructor: Prof. Artem Zvavitch

Problem 1. For each natural number i consider set

$$A_i = \{x \in \mathbb{R} : 0 < x < \frac{1}{i}\}.$$

Please find

- $\bigcap_{i=1}^{10} A_i$
- $\bigcup_{i=1}^{10} A_i$
- $\bigcap_{i=1}^{\infty} A_i$
- $\bigcup_{i=1}^{\infty} A_i$

Problem 2. Prove that if A, B, C are sets, then

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

Problem 3. Let $f : A \rightarrow B$ and let G, H be subsets of B . Using the definition of inverse image of a set under f prove that

$$f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H).$$

Problem 4. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Give an example of surjective function from A to B . Prove that there is no function $f : A \rightarrow B$ such that f is a bijection.

Problem 5. (Extra) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions and let H be a subset of C . Prove that

$$(g \circ f)^{-1}(H) = f^{-1}(g^{-1}(H)).$$