Introduction to Analysis 2(42001/52001)
HW1, due Friday, January 20
Instructor: Prof. Artem Zvavitch

Problem 1. *Use the definition of integral to show that the function* $f(x) : [0,2] \to R$ *such that* $f(x) = 3x$ *for* $x \in [0,1]$ *and* 7 *otherwise is Riemann integrable on* $[0,2]$.

Problem 2. *Consider Riemann integrable function* f *on* $[a,b]$ *such that* $|f(x)| < M$ *for all* $x \in [a,b]$ *show that then*

$$|\int_a^b f| \leq M(b-a).$$

Problem 3. *Consider function* h *defined by* $h(x) = x+1$ *for* $x \in [0,1]$ *rational, and* $h(x) = 0$ *otherwise. Show that* h *is not Riemann integrable on* $[0,1]$.

Problem 4. *Consider a continous function* $f(x) \geq 0$ *such that*

$$\int_a^b f = 0.$$

Prove that $f(x) = 0$ *for all* $x \in [a,b]$.

Problem 5. *Is it possible to remove continuity assumption in the previous problem?*