

**Analysis 1(42001/52001)**  
**Home Work 1, due on Friday August 29.**  
**Instructor: Prof. Artem Zvavitch.**

**Problem 1.** *Prove that*

$$S \cup (T \cap U) = (S \cup T) \cap (S \cup U).$$

**Problem 2.** *Consider a function  $f : A \rightarrow B$ . For a subset  $G \subset B$  define*

$$f^{-1}(G) = \{a \in A : f(a) \in G\}.$$

*Please, prove that*

$$f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H),$$

*where  $G$  and  $H$  are subsets of  $B$ .*

**Problem 3.** *Prove that*

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4},$$

*for all  $n \in \mathbb{N}$ .*

**Problem 4.** *Prove that*

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \geq \sqrt{n},$$

*for all  $n \in \mathbb{N}$ .*

**Problem 5.** *Let  $A$  and  $B$  be finite sets show that*

$$\#(A \cup B) = \#A + \#B - \#(A \cap B),$$

*where  $\#(A)$  denotes the number of elements in  $A$ .*

**Problem 6. (EXTRA 5pts).** *Prove that*

$$\frac{n^n}{3^n} \leq n! \leq \frac{n^n}{2^n},$$

*for all  $n \in \mathbb{N}$ ,  $n \geq 6$ .*