

**Introduction to Analysis 1(42001/52001 Section 01)**

**HW2, due Wednesday, September 14**

**Instructor: Prof. Artem Zvavitch**

**Problem 1.** Consider real numbers  $a_1$  and  $b$ . Let for any  $n \in \mathbb{N}$

$$a_n = a_1 + (n - 1)b.$$

Prove, using the principal of mathematical induction, that

$$a_1 + a_2 + a_3 + \cdots + a_k = \frac{(a_k + a_1)k}{2}.$$

**Problem 2.** Prove that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n},$$

for all  $n \in \mathbb{N}$ .

**Problem 3.** Show that a union of two countable sets is also countable (hint: check the book).

**Problem 4.** Let  $A$  and  $B$  be a countable sets show that  $A \cap B$  is also a countable set.

**Problem 5.** Consider a set  $S$  whose elements are nonoverlapping intervals of length 1 (i.e. for any  $[a_1, b_1] \in S$  and  $[a_2, b_2] \in S$   $[a_1, b_1] \cap [a_2, b_2] = \emptyset$  and  $b_1 - a_1 = b_2 - a_2 = 1$ ). PLEASE SHOW THAT  $S$  is a countable set.