Introduction to Analysis 1(42001/52001)
HW2, due Wednesday, September 16
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that*

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Problem 2. *Prove that*

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n},$$

for all $n \in N$.

Problem 3. (Extra) *Prove that for* $n \geq 5$

$$\frac{n^n}{3^n} \leq n! \leq \frac{n^n}{2^n}.$$

Problem 4. *Consider a set A. We say* $\#A = 0$ *if A is the empty set and we say that* $\#A = n$ *if there is a bijection* $f : A \to \{1, \ldots, n\}$. *We say A is finite if A is empty or* $\#A = n$ *for some natural number n. Please, show that* $\#A$ *is well defined, i.e. for finite set A there is only one number n such that* $\#A = n$. *(hint: You may use a book or any other source*

Problem 5. *Let A and B be a countable sets show that* $A \cap B$ *is also a countable set.*

Problem 6. *Consider a set S whose elements are nonoverlapping intervals of length 1 (i.e. for any* $[a_1, b_1] \in S$ *and* $[a_2, b_2] \in S$ $[a_1, b_1] \cap [a_2, b_2] = \emptyset$ *and* $b_1 - a_1 = b_2 - a_2 = 1$*). PLEASE SHOW THAT S is a countable set.*