Introduction to Analysis 2
Home Work 2, due Wednesday, February 10.
Instructor: Prof. Artem Zvavitch

Problem 1. *Consider an interval $I = [a, b] \subset R$ and a differentiable function f on I. Assume that f' is bounded on I, prove that f is a Lipschitz function.*

Problem 2. *Consider a Lipschitz function f on $I = [a, b] \subset R$ is it true that f is differentiable on I.*

Problem 3. *Find an example of a function which is differentiable on $[-1, 1]$, but not twice differentiable on $[-1, 1]$.*

Problem 4. *Prove that the derivative of the function*

$$f(x) = e^x(x-1)(x-2)(x-3)(x-4)(x-5)$$

has at least four roots.

Problem 5. *Find*

- $\lim_{x\to 0} \frac{\sin x^2}{x}$.
- $\lim_{x\to \infty} \frac{logx}{e^x}$.
- $\lim_{x\to 0+} (\sin x)^x$.

Problem 6. *Show that if $x > 0$, then*

$$1 + \frac{1}{2}x - \frac{1}{8}x^2 \leq \sqrt{1+x} \leq 1 + \frac{1}{2}x.$$

Problem 7. *Show that the sum of two convex functions is again a convex function. Is the same statement true for product? Construct a convex function defined on R which is increasing for all $x \in R$.*