

**Introduction to Analysis 2 (42001/52001)**  
**HW2, due Friday, January 27**  
**Instructor: Prof. Artem Zvavitch**

**Problem 1** *If  $f$  and  $g$  are continuous functions on  $[a, b]$  such that*

$$\int_a^b f = \int_a^b g,$$

*prove that there exists  $c \in [a, b]$  so that  $f(c) = g(c)$  (Yes, you may use results from previous HW).*

**Problem 2** *Show that if  $g(x) = \sin(1/x)$  for  $x \in (0, 1]$  and  $g(0) = 0$  then  $g(x) \in \mathcal{R}[0, 1]$ .*

**Problem 3** *Give an example of function  $f(x)$  on  $[a, b]$  such that  $f(x) \in \mathcal{R}[c, b]$  for all  $c \in (a, b)$  but  $f(x) \notin \mathcal{R}[c, b]$ .*

**Problem 4** *Show that if  $f(x)$  is bounded on  $[a, b]$  and continuous on  $[a, b] \setminus E$ , where  $E$  is a finite set, then  $f \in \mathcal{R}[a, b]$ . Also show that you can not remove the boundness assumption.*

**Problem 5** *Find  $\int_a^b |x| dx$ .*

**Problem 6** *Find  $F'(x)$  if  $F(x) = \int_{x^2}^x \sqrt{1+t^2} dt$ .*