

Introduction to Analysis 1(42001/52001)
Home Work 2, due on Thursday SEPTEMBER 11.
Instructor: Prof. Artem Zvavitch.

Problem 1. *Let A and B be countable sets. Show that $A \cap B$ is countable.*

Problem 2. *Consider a set S whose elements are non-overlapping intervals of length 1 (i.e. for any $[a, b] \in S$ and $[d, c] \in S$ we have that $b - a = c - d = 1$ and $[a, b] \cap [d, c] = \emptyset$). Please, show that S is a countable set.*

Problem 3. *Let S be a set of real numbers x such that x is a solution of a quadratic equation with integer coefficient, i.e.*

$$S = \{x \in \mathbb{R} : nx^2 + mx + k = 0 \text{ for some } n, m, k \in \mathbb{Z}\}.$$

Proof that S is countable.

Problem 4. *Consider $a, b, c \in \mathbb{R}$. Please, prove that*

- $a^2 + b^2 = 0$ if and only if $a = b = 0$.
- If $\frac{1}{ab} > 0$ then either $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$.
- If $b > 0$ and $c > 0$ then $\sqrt{b+c} > \sqrt{c}$.
- For any positive numbers a, b, c, d if $a < b$ and $c < d$, then $ac < bd$.
- Please, prove that $1.4 < \sqrt{2} < 1.5$.
- **Triangle inequality:** $|a + b| \leq |a| + |b|$.