

Introduction to Analysis II
Home Work 2, due Thursday, January 29.
Instructor: Prof. Artem Zvavitch

Problem 1. Find the points of relative extrema

$$f(x) = x|x^2 - 16|$$

on the interval $[-5, 5]$.

Problem 2. Give example of a uniformly continuous but not differentiable function on interval $[-1, 1]$.

Problem 3. Now give an example of a uniformly continuous function on $[0, 1]$ that is differentiable on $(0, 1)$ but whose derivative is not bounded on $(0, 1)$.

Problem 4. Prove that if f is differentiable on the interval $I \subset \mathbb{R}$ and $f'(x) \neq 0$ for all $x \in I$ then either $f'(x) > 0$ for all $x \in I$ or $f'(x) < 0$ for all $x \in I$.

Problem 5. Prove that the derivative of the function

$$f(x) = e^x(x-1)(x-2)(x-3)(x-4)(x-5)$$

has at least four roots.

Problem 6. Consider $a > b > 0$ and let $n \in \mathbb{N}$ satisfy $n \geq 2$. Prove that

$$a^{1/n} - b^{1/n} < (a - b)^{1/n}.$$

Hint: Play with function $f(x) = x^{1/n} - (x-1)^{1/n}$.

Problem 7. Please, find,

- $\lim_{x \rightarrow 0} \frac{\sin x^2}{x}$.
- $\lim_{x \rightarrow \infty} \frac{\log x}{e^x}$.
- $\lim_{x \rightarrow 0^+} (\sin x)^x$.