Introduction to Analysis 1(42001/52001 Section 01)
HW3, due Wednsday, September 23
Instructor: Prof. Artem Zvavitch

Problem 1. *Consider* $a, b, c \in \mathbb{R}$. *Prove*

- $a^2 + b^2 = 0$ *if and only if* $a = b = 0$.
- *If* $\frac{1}{ab} > 0$ *then either* $a > 0$ *and* $b > 0$ *OR* $a < 0$ *and* $b < 0$.
- $-\frac{1}{3} < 0$
- *If* $b > 0$ *and* $c > 0$ *then* $\sqrt{b+c} > \sqrt{c}$.
- *If* $x > 0$ *and* n *is a natural number, then* $(x+1)^n \geq (x+1)$. *Is the same true for* $x < 0$? *Please stay and prove a corrected version of this statement for* $x < 0$.
- **The triangle inequality:** $|a + b| \leq |a| + |b|$.
- *Using above inequality prove:* $|a| - |b| \leq |a - b|$ *and* $|b| - |a| \leq |a - b|$.

Problem 2. *Solve (please, show and explain ALL steps)*

- $\frac{x^2-2x+1}{x-7} > 0$
- $|x + 1| \leq |x - 7|$