Introduction to Analysis 2
Home Work 3, due Monday, February 22.
Instructor: Prof. Artem Zvavitch

Problem 1. *Please construct a function which is infinitely differentiable on R (i.e. $f^{(n)}(x)$ exists for all $n \in N$ and $x \in R$), such that $f^{(n)}(0) = 0$ for all n, but $f(x) \not\equiv 0$. Explain why this example do not contradict Teylor's theorem.*

Problem 2. *Compute e correct to 6 decisimal place.*

Problem 3. *Use the definition of integral to show that function $f(x) = 1$ for $x \in [0,1]$ and $f(x) = 3x$ for $x \in (1,2]$ is integrable on $[0,2]$.*

Problem 4. *Show that the function $f(x) = 0$ for $x = 0$ and $f(x) = 1/x$ for $x \in (0,1]$ is not integrable function on $[0,1]$.*

Problem 5. *Let f be a Riemann integrable function on $[a,b]$. Consider function $g(x)$ on $[a,b]$ such that $g(x) = f(x)$ for all except finite number of points $c_1, c_2, ..., c_n \in [a,b]$. Prove that $g(x)$ is Riemann integrable and*

$$\int_a^b g dx = \int_a^b f dx.$$

Problem 6. *Let f be a Riemann integrable function on $[a,b]$. Consider function $g(x)$ on $[a,b]$ such that $g(x) = f(x)$ for all except COUNTABLE set of points $c_1, c_2, c_3, ... \in [a,b]$. Show that $g(x)$ is not necessary integrable on $[a,b]$.*