Introduction to Analysis 1(42001/52001)
HW3, due TUESDAY, September 23
Instructor: Prof. Artem Zvavitch

Problem 1. *Solve (please, show and explain ALL steps)*

- $-2x + 7 > x - 1$.
- $\frac{x^2-2x+1}{x-7} > 0$.
- $|x+1| \leq |x-7|$.

Problem 2. *Let A and B be subsets of $\mathbb{R}$ such that $A \cap B \neq \emptyset$*

- *Prove that* $\min(\sup(A), \sup(B))$ *is an upper bound for* $A \cap B$.
- *Is it true that* $\min(\sup(A), \sup(B)) = \sup(A \cap B)$. *(if yes then prove it if the answer is No show a counterexample).*

Problem 3. *Find the infimum and the supremum of* $E = \{x \in \mathbb{R} : x = \frac{1}{n} + (-1)^n,$ *for all* $n \in N\}$.

Problem 4. *Consider an nonempty set $E \subset \mathbb{R}$. Prove that E has a supremum if and only if $-E = \{x \in \mathbb{R} : -x \in E\}$ has an infimum, in which case*

$$\inf(-E) = -\sup(E).$$

Problem 5. *Suppose that f and g are real valued functions with common domain $D \subset \mathbb{R}$. PLEASE, check the folloving statements (if it is true then prove it, if it is false then show a counterexample)*

- $\sup_{x\in D} f(x) + \sup_{x\in D} g(x) \leq \sup_{x\in D}(f(x) + g(x))$.
- $\sup_{x\in D} f(x) + \sup_{x\in D} g(x) = \sup_{x\in D}(f(x) + g(x))$.
- $\sup_{x\in D} f(x) \times \sup_{x\in D} g(x) \leq \sup_{x\in D}(f(x) \times g(x))$.
- $\inf_{x\in D} f(x) \times \inf_{x\in D} g(x) \leq \inf_{x\in D}(f(x) \times g(x))$.

Problem 6. (Extra 10pts). *Prove the genral Arithmetic-Geometric Mean Inequality: $a_1, a_2, \ldots, a_n$ are positive numbers, then*

$$(a_1 \cdot a_2 \cdot \cdots \cdot a_n)^{\frac{1}{n}} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

Hints: *Yes you can use any book you wish, but then you MUST prove everything that you used (if it was not proved in class). One of the ideas, consider $n = 2^k$ and prove the inequality using incduction on k (i.e., you will get the result when n is a power of 2). To prove it for other n use the fact that if the inequality is true for $n+1$, then it is true for n (prove it!).*