Introduction to Analysis II
Home Work 3, due Thursday, February 5.
Instructor: Prof. Artem Zvavitch

Problem 1. *Find*

- $\lim\limits_{x\to\infty} \frac{xe^x}{e^x+\ln x}$
- $\lim\limits_{x\to 0} \frac{e^x \sin x}{x^2+1}$
- $\lim\limits_{x\to 0+} x^{2x}$
- $\lim\limits_{x\to\infty} x^{1/x}$
- $\lim\limits_{x\to 0+} (\frac{1}{x} - \frac{1}{\arctan x})$

Next problems are connected with Taylors theorem, think about them, but you may wait till Tuesday for final attack!

Problem 2. *Prove that*

$$1 - \frac{x^2}{2} \le cosx \le 1 - \frac{x^2}{2} + \frac{x^4}{24}.$$

Problem 3. *Show that the sum of two convex functions is again a convex function. Is the same statement true for product? Construct a convex function defined on* $\mathbb{R}$ *which is increasing for all* $x \in \mathbb{R}$

Problem 4. *Compute* e *correct to* 6 *decimal place.*

Problem 5. (Extra 10 points) *Please construct a function which is infinitely differentiable on* $\mathbb{R}$ *(i.e.* $f^{(n)}(x)$ *exists for all* $x \in \mathbb{R}$ *and* $n \in \mathbb{N}$*), such that* $f^{(n)}(0) = 0$ *for all* n*, but* $f(x) \neq 0$ *for all* $x \neq 0$*. Explain why this example do not contradict Teylor's theorem.*