

**Introduction to Analysis 1(42001/52001 Section 01)**  
**HW4, due Wednesday, September 28**  
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** Let  $A$  and  $B$  be nonempty subspaces of  $\mathbb{R}$  such that  $A \cap B \neq \{0\}$

- Prove that  $\min(\sup(A), \sup(B))$  is an upper bound for  $A \cap B$ .
- Is it true that  $\min(\sup(A), \sup(B)) = \sup(A \cap B)$ . (if yes then prove it if the answer is No show a counterexample).

**Problem 2.** Suppose that  $f$  and  $g$  are real valued functions with common domain  $D \subset \mathbb{R}$ . PLEASE, check the following statements (if it is true then prove it if it is false then show a counterexample)

- $\sup_{x \in D} f(x) + \sup_{x \in D} g(x) \leq \sup_{x \in D} (f(x) + g(x))$ .
- $\sup_{x \in D} f(x) + \sup_{x \in D} g(x) = \sup_{x \in D} (f(x) + g(x))$ .
- $\sup_{x \in D} f(x) \times \sup_{x \in D} g(x) \leq \sup_{x \in D} (f(x) \times g(x))$ .
- $\inf_{x \in D} f(x)^2 \leq \left( \inf_{x \in D} (f(x)) \right)^2$ .