

Introduction to Analysis 1(42001/52001)
HW4, due Wednesday, September 30
Instructor: Prof. Artem Zvavitch

Problem 1. Let A and B be nonempty subsets of \mathbb{R} such that $A \cap B \neq \emptyset$

- Prove that $\min(\sup(A), \sup(B))$ is an upper bound for $A \cap B$.
- Is it true that $\min(\sup(A), \sup(B)) = \sup(A \cap B)$. (if yes then prove it if the answer is No show a counterexample).

Problem 2. Find the infimum and the supremum of $E = \{x \in \mathbb{R} : x = \frac{1}{n} + (-1)^n, \text{ for all } n \in \mathbb{N}\}$.

Problem 3. Consider an nonempty set $E \subset \mathbb{R}$. Prove that E has a supremum if and only if $-E = \{x \in \mathbb{R} : -x \in E\}$ has an infimum, in which case

$$\inf(-E) = -\sup(E).$$

Problem 4. Suppose that f and g are real valued functions with common domain $D \subset \mathbb{R}$. PLEASE, check the following statements (if it is true then prove it, if it is false then show a counterexample)

- $\sup_{x \in D} f(x) + \sup_{x \in D} g(x) \leq \sup_{x \in D} (f(x) + g(x))$.
- $\sup_{x \in D} f(x) + \sup_{x \in D} g(x) = \sup_{x \in D} (f(x) + g(x))$.
- $\sup_{x \in D} f(x) \times \sup_{x \in D} g(x) \leq \sup_{x \in D} (f(x) \times g(x))$.
- $\inf_{x \in D} f(x) \times \inf_{x \in D} g(x) \leq \inf_{x \in D} (f(x) \times g(x))$.

Problem 5. (Extra 10pts). Prove the genral Arithmetic-Geometric Mean Inequality: a_1, a_2, \dots, a_n are positive numbers, then

$$(a_1 \cdot a_2 \cdot \dots \cdot a_n)^{\frac{1}{n}} \leq \frac{a_1 + a_2 + \dots + a_n}{n}.$$

Hints: Yes you can use any book you wish, but then you MUST prove everything that you used (if it was not proved in class). One of the ideas, consider $n = 2^k$ and prove the inequality using induction on k (i.e., you will get the result when n is a power of 2). To prove it for other n use the fact that if the inequality is true for $n + 1$, then it is true for n (prove it!).