**Introduction to Analysis 1(42001/52001)**
**HW4, due Monday, October 3**
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** *Let $A$ and $B$ be subsets of $\mathbb{R}$ such that $A \cap B \neq \emptyset$*

- *Prove that* $\min(\sup(A), \sup(B))$ *is an upper bound for* $A \cap B$.
- *Is it true that* $\min(\sup(A), \sup(B)) = \sup(A \cap B)$. *(if yes then prove it if the answer is No show a counterexample).*

**Problem 2.** *Find the infimum and the supremum of* $E = \{x \in \mathbb{R} : x = \frac{1}{n} + (-1)^n,\ \text{for all } n \in N\}$.

**Problem 3.** *Consider an nonempty set $E \subset \mathbb{R}$. Prove that $E$ has a supremum if and only if $-E = \{x \in \mathbb{R} : -x \in E\}$ has an infimum, in which case*

$$\inf(-E) = -\sup(E).$$

**Problem 4.** *Suppose that $f$ and $g$ are real valued functions with common domain $D \subset \mathbb{R}$. PLEASE, check the folloving statements (if it is true then prove it, if it is false then show a counterexample)*

- $\sup_{x \in D} f(x) + \sup_{x \in D} g(x) \leq \sup_{x \in D}(f(x) + g(x))$.
- $\sup_{x \in D} f(x) + \sup_{x \in D} g(x) = \sup_{x \in D}(f(x) + g(x))$.
- $\sup_{x \in D} f(x) \times \sup_{x \in D} g(x) \leq \sup_{x \in D}(f(x) \times g(x))$.
- $\inf_{x \in D} f(x) \times \inf_{x \in D} g(x) \leq \inf_{x \in D}(f(x) \times g(x))$.

**Problem 5. (Extra 10pts)**. *Prove the genral Arithmetic-Geometric Mean Inequality: $a_1, a_2, \ldots, a_n$ are positive numbers, then*

$$(a_1 \cdot a_2 \cdot \cdots \cdot a_n)^{\frac{1}{n}} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

**Hints:** *Yes you can use any book you wish, but then you MUST prove everything that you used (if it was not proved in class). One of the ideas, consider $n = 2^k$ and prove the inequality using incduction on $k$ (i.e., you will get the result when $n$ is a power of 2). To prove it for other $n$ use the fact that if the inequality is true for $n+1$, then it is true for $n$ (prove it!).*