Introduction to Analysis 2
Home Work 4, due Wednesday, February 22.
Instructor: Prof. Artem Zvavitch

Problem 1. *Show that the sequence* $f_n(x) = n^2x^2e^{-nx}$ *converges uniformly on* (a, ∞), *for any* $x \geq 0$, *but it does not converge uniformly on* $[0, \infty)$.

Problem 2. *Prove that if sequences* f_n, g_n *converge uniformly on* A *to* f *and* g, *resectively, then* $f_n + g_n$ *converges uniformly on* A *to* $f + g$.

Problem 3. *Show that the product of uniformly convergent sequences is not necessary uniformly convergent.* **Hint:** *consider* $f_n = x + \frac{1}{n}$ *and* f_n^2 *on* $\mathbb{R}$.

Problem 4. *Now let's repair the product property: Prove that if sequences of* **bounded functions** f_n, g_n *converge uniformly on* A *to* f *and* g, *resectively, then* f_ng_n *converges uniformly on* A *to* fg.

Problem 5. *Show that the sequence* $x^n/(1 + x^n)$ *dooes not converge uniformly on* $[0, 2]$ *by showing that the limit function is not continuous on* $[0, 2]$.

Problem 6. *Let* $f_n(x) = e^{-nx}/n$ *for* $x \geq 0$. *Study the relation between* $\lim f_n$ *and* $\lim f_n'$.