Introduction to Analysis 1(42001/52001 Section 01)
HW5, due Monday, October 19
Instructor: Prof. Artem Zvavitch

Problem 1. *Use properties of convergent sequences to find the limits below. AFTER, use the definition to show that*

- $\lim\limits_{n\to\infty} \frac{2(-1)^n}{2-n}$.
- $\lim\limits_{n\to\infty} \frac{n+1}{n+2}$.
- $\lim\limits_{n\to\infty} \frac{n}{n^2+\cos n}$.

Problem 2. *Show an example of two convergent sequences* $(a_n : n \in N)$ *and* $(b_n : n \in N)$, *such that* $a_n > b_n$ *for all* $n \in N$, *but*

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$$

Problem 3. *Please, find an example of two sequences* $(a_n : n \in N)$ *and* $(b_n : n \in N)$ *so that* $(a_n : n \in N)$ *and* $(b_n : n \in N)$ *are divergent but* $(a_n + b_n : n \in N)$ *is convergent.*

Problem 4. *Show that if* $(a_n : n \in N)$ *is a convergent seguence than* $a_{n+1} - a_n$ *converges to zero.*

Problem 5. *Show example of a DIVERGENT sequence* $(a_n : n \in N)$ *such that* $a_{n+1} - a_n$ *converges to zero.*

Problem 6. *Consider a sequence* $(a_n : n \in N)$, *such that*

$$|a_{n+1} - a_n| \le \frac{1}{3^n}.$$

Show that $(a_n : n \in N)$ *is convergent.*