Introduction to Analysis 2
Home Work 5+6, due MONDAY, March 22.
Instructor: Prof. Artem Zvavitch

Problem 1. *Show that the function*

$$f(x)=\int_0^x e^{-t^2}dt$$

is a 1*-Lipschitz function (i.e. for any $x, y \in R$ we have $|f(x)-f(y)| \le |x-y|$).*

Problem 2. *Find $f(x)$ if*

$$\int\limits_0^x f(t)dt=\int\limits_0^{2x} f(t)dt,$$

for all $x \in R$ and $f(x)$ is a continuous function.

Problem 3. *Find $F'(x)$ if*

$$F(x)=\int\limits_{x^2}^{e^x} t\cos t dt$$

Problem 4. *Find*

$$\int\limits_{-1}^{1} t\cos(t^2-1)dt$$

Problem 5. *Use Lebesgue's Integrability criterion to show that the function $f(x)=\frac{sinx}{x}$ $x \neq 0$ and $f(0)=0$ is integrable on $[-1,1]$.*

Problem 6. *Let $f, g \in \mathbb{R}[a,b]$.*

- *If $t \in R$, show that $\int_a^b (tf \pm g)^2 \ge 0$.*
- *Use the previous fact to show that $2|\int_a^b (fg)| \le t\int_a^b f^2+\frac{1}{t}\int_a^b g^2$ for $t>0$.*
- *If $\int_a^b f^2=0$, show that $\int_a^b fg=0$.*
- *Now provet that $|\int_a^b (fg)|^2 \le \left(\int_a^b |fg|\right)^2 \le \left(\int_a^b |f|^2\right)\left(\int_a^b |g|^2\right)$. This inequality is called the* **Cauchy-Bunyakowski-Schwartz Inequality***.*
- *Now prove the triangle inequality for functions:*

$$\sqrt{\int_a^b (f-g)^2dx} \le \sqrt{\int_a^b f^2dx}+\sqrt{\int_a^b g^2dx}.$$