Introduction to Analysis 1(42001/52001 Section 01)
HW5, due Monday, October 19
Instructor: Prof. Artem Zvavitch

Problem 1. *Use properties of convergent sequences to find the limits below. AFTER, use the definition to show that*

- $\lim_{n\to\infty} \frac{2(-1)^n}{2-n}$.
- $\lim_{n\to\infty} \frac{n+1}{n+2}$.
- $\lim_{n\to\infty} \frac{n}{n^2+\cos n}$.

Problem 2. *Show an example of two convergent sequences $(a_n : n \in N)$ and $(b_n : n \in N)$, such that $a_n > b_n$ for all $n \in N$, but*

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$$

Problem 3. *Please, find an example of two sequences $(a_n : n \in N)$ and $(b_n : n \in N)$ so that $(a_n : n \in N)$ and $(b_n : n \in N)$ are divergent but $(a_n + b_n : n \in N)$ is convergent.*

Problem 4. *Please, find an example of two sequences $(a_n : n \in N)$ and $(b_n : n \in N)$ so that $(a_n : n \in N)$ and $(b_n : n \in N)$ are divergent but $(a_n \times b_n : n \in N)$ is convergent.*

Problem 5. *Show that if $(a_n : n \in N)$ is a convergent seguence than $a_{n+1} - a_n$ converges to zero.*

Problem 6. *Show example of a DIVERGENT sequence $(a_n : n \in N)$ such that $a_{n+1} - a_n$ converges to zero.*

Problem 7. *Consider a sequence $(a_n : n \in N)$, such that*

$$|a_{n+1} - a_n| \leq \frac{1}{3^n}.$$

Show that $(a_n : n \in N)$ is convergent.