

Introduction to Analysis 1(42001/52001)
HW5, due Thursday, October 9
Instructor: Prof. Artem Zvavitch

Problem 1. *Guess the limits and use the definition to prove it.*

- $\lim_{n \rightarrow \infty} \frac{2(-1)^n}{2-n}$.
- $\lim_{n \rightarrow \infty} \frac{n+1}{n+2}$.
- $\lim_{n \rightarrow \infty} \frac{n}{n^2 + \cos n}$.

Problem 2. *Consider a sequence $a_{2n} = 1$ and $a_{2n-1} = \frac{1}{2n-1}$, for $n \in \mathbb{N}$. Is this sequence convergent or divergent?*

Problem 3. *Show an example of two convergent sequences $(a_n : n \in \mathbb{N})$ and $(b_n : n \in \mathbb{N})$, such that $a_n > b_n$ for all $n \in \mathbb{N}$, but*

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n$$

Problem 4. *Please, find an example of two sequences $(a_n : n \in \mathbb{N})$ and $(b_n : n \in \mathbb{N})$ so that $(a_n : n \in \mathbb{N})$ and $(b_n : n \in \mathbb{N})$ are divergent but $(a_n + b_n : n \in \mathbb{N})$ is convergent.*

Problem 5. *Please, find an example of two sequences $(a_n : n \in \mathbb{N})$ and $(b_n : n \in \mathbb{N})$ so that $(a_n : n \in \mathbb{N})$ and $(b_n : n \in \mathbb{N})$ are divergent but $(a_n \times b_n : n \in \mathbb{N})$ is convergent.*

Problem 6. *Show that if $(a_n : n \in \mathbb{N})$ is a convergent sequence then $a_{n+1} - a_n$ converges to zero.*

Problem 7. *Show example of a DIVERGENT sequence $(a_n : n \in \mathbb{N})$ such that $a_{n+1} - a_n$ converges to zero.*

Problem 8. *Consider a sequence $(a_n : n \in \mathbb{N})$, such that*

$$|a_{n+1} - a_n| \leq \frac{1}{3^n}.$$

Show that $(a_n : n \in \mathbb{N})$ is convergent.