

Introduction to Analysis II
Home Work 5, due Tuesday, February 24 .
Instructor: Prof. Artem Zvavitch

Problem 1. Use the definition of the Riemann integral to show that the function $f(x) : [0, 2] \rightarrow \mathbb{R}$ such that $f(x) = 3x$ for $x \in [0, 1]$ and 7 otherwise is Riemann integrable on $[0, 2]$.

Problem 2. Consider Riemann integrable function f on $[a, b]$ such that $|f(x)| < M$ for all $x \in [a, b]$ show that then

$$\left| \int_a^b f dx \right| \leq M(b - a).$$

Problem 3. Consider function h defined by $h(x) = x + 1$ for $x \in [0, 1]$ rational, and $h(x) = 0$ otherwise. Show that h is not Riemann integrable on $[0, 1]$.

Problem 4. Consider a continuous function $f(x) \geq 0$ such that

$$\int_a^b f = 0.$$

Prove that $f(x) = 0$ for all $x \in [a, b]$.

Problem 5. Is it possible to remove continuity assumption in the previous problem?