

Introduction to Analysis 1(42001/52001 Section 01)
HW6, due Wednesday, October 28
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that the following sequences are convergent and find the limits (you may use ANY results from class/book)*

(1) $b_1 = 8, b_{n+1} = \frac{1}{2}b_n + 2.$

(2) $\left(1 + \frac{1}{n}\right)^{n+1}.$

(3) $\left(1 - \frac{1}{n}\right)^n.$

(4) $\left(1 + \frac{1}{n^3}\right)^{4n^3}.$

(5) $\left(1 + \frac{1}{2n}\right)^{7n}.$

Problem 2. *Prove that the following sequence is convergent*

$$a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}.$$

Problem 3. *Consider a sequence $(a_n : n \in \mathbb{N})$, such that*

$$|a_{n+1} - a_n| \leq \frac{1}{3^n}.$$

Show that $(a_n : n \in \mathbb{N})$ is convergent.

Problem 4. *Assume $a_1 < a_2$ are arbitrary real numbers and $a_n = \frac{1}{2}(a_{n-2} + a_{n-1})$, for $n > 2$, show that a_n is a Cauchy sequence. Find the limit.*

Problem 5. *Is it true that every bounded sequence is Cauchy sequence?
Is it true that every bounded sequence has a Cauchy subsequence?*