Introduction to Analysis 1(42001/52001 Section 01)
HW6, due Monday, October 31
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that the following sequences are convergent and find the limits (you may use ANY results from class/book)*

(1) $b_1 = 8$, $b_{n+1} = \frac{1}{2}b_n + 2$.
(2) $\left(1 + \frac{1}{n}\right)^{n+1}$.
(3) $\left(1 - \frac{1}{n}\right)^{n}$.
(4) $\left(1 + \frac{1}{n^3}\right)^{4n^3}$.
(5) $\left(1 + \frac{1}{2n}\right)^{7n}$.

Problem 2. *Please, decide if the following sequences are Couchy or not (do not forget to provide an explanation!):*

(1) $\frac{(-1)^n}{2^n}$.
(2) $(\cos \frac{\pi}{2} n) \times n$.
(3) $\sqrt{n}$.

Problem 3. *Consider a sequence x_n such that*

$$\lim_{n\to\infty} |x_n - x_{n-1}| = 0.$$

Is it true that x_n then must be a Cauchy sequence? Bounded Sequence? Convergent Sequence? (Hint: check the last sequence in the previous problem.)

Problem 4. *Prove that the following sequence is convergent*

$$a_n = 1 + \frac{3}{2!} + \frac{3^2}{3!} + \cdots + \frac{3^{n-1}}{n!}.$$

Problem 5. *Consider a sequence $(a_n : n \in N)$, such that*

$$|a_{n+1} - a_n| \leq \frac{1}{3^n}.$$

Show that $(a_n : n \in N)$ is convergent.

Problem 6. *Assume $a_1 < a_2$ are arbitrary real numbers and $a_n = \frac{1}{2}(a_{n-2} + a_{n-1})$, for $n > 2$, show that a_n is a Cauchy sequence. Find the limit.*

Problem 7. *Is it true that every bounded sequence is Cauchy sequence? Is it true that every bounded sequence has a Cauchy subsequence?*