

Introduction to Analysis 1(42001/52001 Section 01)
HW6, due Thursday, October 23
Instructor: Prof. Artem Zvavitch

Problem 1. *Prove that the following sequences are convergent and find the limits (you may use ANY results from class/book)*

(1) $b_1 = 8, b_{n+1} = \frac{1}{2}b_n + 2.$

(2) $\left(1 + \frac{1}{n}\right)^{n+1}.$

(3) $\left(1 - \frac{1}{n}\right)^n.$

(4) $\left(1 + \frac{1}{n^3}\right)^{4n^3}.$

(5) $\left(1 + \frac{1}{2n}\right)^{7n}.$

Problem 2. *Let (x_n) be a bounded sequence of real numbers. Assume $S = \sup x_n$ and $s = \inf x_n$, prove that there is a subsequence of (x_n) convergent to S and another subsequence convergent to s .*

Problem 3. *Give the example of sequence which does not contain a convergent subsequence.*

Problem 4. *Please, decide if the following sequences are Cauchy or not (do not forget to provide an explanation!):*

(1) $\frac{(-1)^n}{2^n}.$

(2) $\left(\cos \frac{\pi}{2}n\right) \times n.$

(3) $\sqrt{n}.$

Problem 5. *Consider a sequence x_n such that*

$$\lim_{n \rightarrow \infty} |x_n - x_{n-1}| = 0.$$

Is it true that x_n then must be a Cauchy sequence? Bounded Sequence? Convergent Sequence? (Hint: check the last sequence in the previous problem.)

Problem 6. *Prove that the following sequence is convergent*

$$a_n = 1 + \frac{3}{2!} + \frac{3^2}{3!} + \cdots + \frac{3^{n-1}}{n!}.$$

Problem 7. *Assume $a_1 < a_2$ are arbitrary real numbers and $a_n = \frac{1}{2}(a_{n-2} + a_{n-1})$, for $n > 2$, show that a_n is a Cauchy sequence. Find the limit.*

Problem 8. *Is it true that every bounded sequence is Cauchy sequence? Is it true that every bounded sequence has a Cauchy subsequence?*