Introduction to Analysis II (42001/52001)
HW6, due Thursday, March 5
Instructor: Prof. Artem Zvavitch

Problem 1 *If f and g are continuous functions on $[a, b]$ such that*

$$\int_a^b f = \int_a^b g,$$

prove that there exists $c \in [a, b]$ so that $f(c) = g(c)$. Show an example proving that you can not remove the assumption of continuity

Problem 2 *Show that if $g(x) = \sin(1/x)$ for $x \in (0, 1]$ and $g(0) = 0$ then $g(x) \in \mathcal{R}[0, 1]$.*

Problem 3 *Show that for any continuous function $f : [0, 1] \to \mathbb{R}$:*

$$\int_{-1}^1 f(x^2)dx = 2\int_0^1 f(x^2)dx.$$

Problem 4 *Give an example of function $f(x)$ on $[a, b]$ such that $f(x) \in \mathcal{R}[c, b]$ for all $c \in (a, b)$ but $f(x) \notin \mathcal{R}[a, b]$.*

Problem 5 *Show that if $f(x)$ is bounded on $[a, b]$ and continuous on $[a, b]\backslash E$, where E is a finite set, then $f \in \mathcal{R}[a, b]$. Also show that you can not remove the boundness assumption.*

Problem 6 *Proof or find counterexample to the following statement: if $f(x) : [0, 1] \to \mathbb{R}$, such that $f(x) > 0$ and $f \in \mathcal{R}[0, 1]$, then $1/f \in \mathcal{R}[0, 1]$.*

Problem 7 *Find $\int_a^b |x|dx$.*

Problem 8 *Find $F'(x)$ if $F(x) = \int_{x^2}^x \sqrt{1 + t^2}dt$.*

Problem 9 *Compute*

$$\int_0^1 e^{-t^2/2}tdt.$$