

**Analysis 1(42001/52001)**  
**Home Work 6, due on Friday October 22.**  
**Instructor: Prof. Artem Zvavitch.**

**Problem 1.** Let  $(x_n)$  be a bounded sequence of real numbers. Consider a sequence  $b_n = \max_{k=1, \dots, n} x_k$ . Prove that  $b_n$  is convergent.

**Problem 2.** Give the example of sequence which does not contain a convergent subsequence.

**Problem 3.** Please, decide if the following sequences are Cauchy or not (do not forget to provide an explanation!):

- (1)  $\frac{(-1)^n}{2^n}$ .
- (2)  $(\cos \frac{\pi}{2}n) \times n$ .
- (3)  $\sqrt{n}$ .
- (4)  $\sum_{k=1}^n \frac{(-1)^k}{3^k}$ .
- (5)  $\sum_{k=1}^n \frac{x^k}{k!}$ , where  $x \in \mathbb{R}$ .

**Problem 4.** Consider a sequence  $x_n$  such that

$$\lim_{n \rightarrow \infty} |x_n - x_{n-1}| = 0.$$

Is it true that  $x_n$  then must be a Cauchy sequence? Bounded Sequence? Convergent Sequence?

**Problem 5.** Prove that the following sequence is convergent

$$a_n = 1 + \frac{3}{2!} + \frac{3^2}{3!} + \dots + \frac{3^{n-1}}{n!}.$$

**Problem 6.** Assume  $a_1 < a_2$  are arbitrary real numbers and  $a_n = \frac{1}{2}(a_{n-2} + a_{n-1})$ , for  $n > 2$ , show that  $a_n$  is a Cauchy sequence. Find the limit.

**Problem 7.** Is it true that every bounded sequence is Cauchy sequence? Is it true that every bounded sequence has a Cauchy subsequence?

**Problem 8.** Prove that the following sequences are convergent and find the limits (you may use ANY results from class/book)

- (1)  $b_1 = 8, b_{n+1} = \frac{1}{2}b_n + 2$ .
- (2)  $(1 + \frac{1}{n})^{n+1}$ .
- (3)  $(1 - \frac{1}{n})^n$ .
- (4)  $(1 + \frac{1}{n^3})^{4n^3}$ .
- (5)  $(1 + \frac{1}{2n})^{7n}$ .