

Introduction to Analysis 2
Home Work 7 and 8, due Monday, April 21. 25 Points
Instructor: Prof. Artem Zvavitch

Problem 1. Consider functions $f_n(x) = e^{-nx^2}$. Find function $f(x)$ such that $f_n(x)$ converges to $f(x)$. Show that $f_n(x)$ does not converge uniformly to $f(x)$.

Problem 2. Assume that $f_n(x)$ converges uniformly to $f(x)$ on $[a, b]$, prove that

$$\lim_{n \rightarrow \infty} \int_a^b |f(x) - f_n(x)| = 0.$$

Show that converse is not true.

Problem 3. Consider a sequence of functions $f_n(x)$, such that each $f_n(x)$ is a bounded function: if $f_n(x)$ converges pointwise to $f(x)$ is it true that $f(x)$ is a bounded function? if $f_n(x)$ converges uniformly to $f(x)$ is it true that $f(x)$ is a bounded function?

Problem 4. Find a sequence of continuous functions convergent pointwise to function $f(x) = \text{sign}(x)$ ($f(x) = 1$, is $x > 0$, $f(x) = -1$, is $x < 0$ and $f(0) = 0$).

Problem 5. PROVE that if $f(x)$ is a function from R to R such that $f(x) = f'(x)$ then there exist a constant C so that $f(x) = Ce^x$.

Problem 6. Use theorem 8.4.6 to find a function $f(x)$ such that $f''(x) = -f(x)$ and $f(0) = f'(0) = 1$.

Problem 7. Show that the function $C(x) = \cos x$ is monotone on $[0, \pi]$. Find the derivative of inverse function for $C(x)$.

Problem 8. Solve the equation $\sin x = x$. Can you solve $\cos x = 1 - \frac{x^2}{2}$?

Problem 9. Please, rewrite theorem 8.4.6 for equation $f''(x) = f(x)$ (hint you need to show that $f(x) = \alpha e^x + \beta e^{-x}$).