

**Introduction to Analysis 1**  
**HW7, due Wednesday, November 7**  
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** Show that  $\sum_{i=1}^n \frac{(-1)^n}{\sqrt{n}}$  is convergent.

**Problem 2.** Use the Cauchy Test to show that  $\sum \frac{1}{n \log n}$  is divergent.

**Problem 3.** Use  $\varepsilon - \delta$ -definition of limit to show that

- $\lim_{x \rightarrow 1} (x^2 + 1) = 2$
- $\lim_{x \rightarrow 1} \frac{x+1}{2-x} = 2$
- $\lim_{x \rightarrow 1} \frac{x^2-1}{1-x} = -2$

**Problem 4.** Use sequential criterion to show that

$$\lim_{x \rightarrow 0} \frac{1}{x^2}$$

does not exist.

**Problem 5.** Use sequential criterion to show that

$$\lim_{x \rightarrow 0} \cos\left(\frac{1}{x}\right)$$

does not exist.

**Problem 6.** Use  $\varepsilon - \delta$ -definition of limit to show that

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$$

**Problem 7.** Use  $\varepsilon - \delta$ -definition of limit to show that

$$\lim_{x \rightarrow c} x^4 = c^4.$$

**Problem 8.** Use  $\varepsilon - \delta$ -definition of limit to show that if

$$\lim_{x \rightarrow c} f(x) = L$$

then

$$\lim_{x \rightarrow c} f(x)^2 = L^2.$$

Show that converse statement is no longer true.