Introduction to Analysis
Exam for fun 2
Instructor: Prof. Artem Zvavitch
Extra 20 points, if submitted on the day of Second exam!

Problem 1. *Check if sequences are convergent or divergent (you may use ANY results from class/book)*

(1) $\left(1+\frac{1}{n}\right)^{n^2}$.
(2) $\left(1+\frac{1}{n^3}\right)^{n}$.
(3) $\left(1-\frac{1}{2n}\right)^{-7n}$.
(4) $\frac{\cos n}{n}$

Problem 2. *Let x_n be a sequence of real numbers such that*

$$|x_{n+1}| \le \frac{1}{3}|x_n|.$$

Show that x_n is a convergent sequence.

Problem 3. *Let $x_1 = 0$ and $x_{n+1} = \frac{1}{10}(x_n + 2)$ show that a) x_n is bounded. b) Monotone (increasing) c) convergent. Finally find the limit of x_n.*

Problem 4. *Let $a_1 = -\frac{1}{2}$ and*

$$a_n = -\frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots + \frac{(-1)^n}{2^n}, \text{ for } n \in N.$$

Show that a_n is a convergent sequence.

Problem 5. *Assume x_n is a bounded increasing sequence of real numbers. Show that $\lim x_n = \sup x_n$.*

Problem 6. *Let x_n be unbounded sequence of negative numbers. Is it true that $\lim_{n\to\infty} x_n = -\infty$? Show that there exits a subsequence x_{n_k} so that $\lim_{k\to\infty} x_{n_k} = -\infty$?*