

Introduction to Analysis 1
HW8, due Wednesday, November 18
Instructor: Prof. Artem Zvavitch

Problem 1. Use $\varepsilon - \delta$ -definition of limit to show that

- $\lim_{x \rightarrow 1} (x^2 + 1) = 2$
- $\lim_{x \rightarrow 1} \frac{x+1}{2-x} = 2$
- $\lim_{x \rightarrow 1} \frac{x^2-1}{1-x} = 2$

Problem 2. Use sequential criterion to show that

$$\lim_{x \rightarrow 0} \frac{1}{x^2}$$

does not exist.

Problem 3. Use sequential criterion to show that

$$\lim_{x \rightarrow 0} \cos\left(\frac{1}{x}\right)$$

does not exist.

Problem 4. Use $\varepsilon - \delta$ -definition of limit to show that

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$$

Problem 5. Use $\varepsilon - \delta$ -definition of limit to show that

$$\lim_{x \rightarrow c} x^4 = c^4.$$

Problem 6. Use $\varepsilon - \delta$ -definition of limit to show that if

$$\lim_{x \rightarrow c} f(x) = L$$

then

$$\lim_{x \rightarrow c} f(x)^2 = L^2.$$

Show that converse statement is no longer true.