

**Introduction to Analysis**  
**Home Work 9, due Wednesday, December 7.**  
**Instructor: Prof. Artem Zvavitch**

**Problem 1.** *Give an example of a function which is continuous on  $I = (0, 1)$ , but unbounded on  $I$ . Prove that ANY UNIFORMLY continuous function on  $I$  is bounded.*

**Problem 2.** *If  $f$  and  $g$  are increasing function on  $[a, b]$  what can you say about  $f + g$ ?  $f \cdot g$ ?*

**Problem 3.** *Consider  $f : [0, 1] \rightarrow \mathbb{R}$  such that  $f$  is continuous,  $f(0) \leq f(1)$  and  $f(x) \neq f(y)$  for all  $x \neq y \in [0, 1]$ . Prove that then  $f$  is increasing.*

**Problem 4.** *Check if the function  $f(x) = x \cdot |x|$  is defferentiable at 0.*

**Problem 5.** *Prove that*

$$|\sin x - \sin y| \leq |x - y|, \text{ for all } x, y \in \mathbb{R}.$$