

Introduction to Analysis
Home Work 9, due Tuesday, November 25.
Instructor: Prof. Artem Zvavitch

Problem 1. Give an example of a function that has a right-hand side limit but not a left-hand limit at a $x = 3$

Problem 2. Suppose that f and g has a limit as $x \rightarrow \infty$ and that $f(x) \leq g(x)$ for all $x \in (15, \infty)$. Prove that

$$\lim_{x \rightarrow \infty} f(x) \leq \lim_{x \rightarrow \infty} g(x).$$

Problem 3. Use $\varepsilon - \delta$ definition to show the continuity of function $f(x) = \frac{1}{x+1}$ at point $x = 2$.

Problem 4. Suppose that f is a continuous function on $[0, 1]$ such that $f(r) = 0$ for every rational number r . Prove that $f(x) = 0$ for all $x \in [0, 1]$.

Problem 5. Suppose that f and g are continuous function on $[0, 1]$ such that $f(r) = g(r)$ for every rational number r . Prove that $f(x) = g(x)$ for all $x \in [0, 1]$.

Problem 6. Show that the function

$$f(x) = \frac{\sqrt{x+1}}{\sqrt{1+\sqrt{x}}}$$

is a continuous function for $x \in [0, \infty)$.

Problem 7. Show an example of bounded set A and a function f continuous on A , such that f is unbounded function on A .

Problem 8. Prove that if

$$|f(x) - f(y)| \leq |x - y|^2$$

for all x, y in $[0, 1]$ then $f(x)$ is a continues function on $[0, 1]$.

Problem 9. Consider a continuous function $f : [0, 1] \rightarrow \mathbb{R}$, such that $f(x) > 0$, for all $x \in [0, 1]$. Prove that there exists a number $\alpha > 0$ such that $f(x) > \alpha$.

Problem 10. Show that function $f(x) = 7^x + \sqrt{x} - 10$ has a root in the interval $[1, 2]$.

Problem 11. Consider a continuous function $f : [0, 1] \rightarrow [0, 1]$, show that the equation $f(x) = x$ has at least one solution.

Problem 12. Give an example of a continuous function $f : [0, \infty) \rightarrow [0, \infty)$, such that the equation $f(x) = x$ has no solutions.